The formula of volumes.
Volumes of pyramids of components Element of Siro

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As it has been proved in work of an **element of Siro [1]** the volume of an elementary tetrahedron at length of edge K is equal $\frac{K^3\sqrt{2}}{12}$.

Throughout all the article we will consider an elementary tetrahedron and an element of Siro as the figures together making an octahedron as it has been shown in work **[1]**.

Continuing to study properties of an element of Siro, see figure 1, we will consider from what elementary volumes this figure consists.

I offer a trihedral pyramid as elementary volume. The element of Siro can be broken into trihedral pyramids in different ways.

I offer a splitting variant shown on figure 1

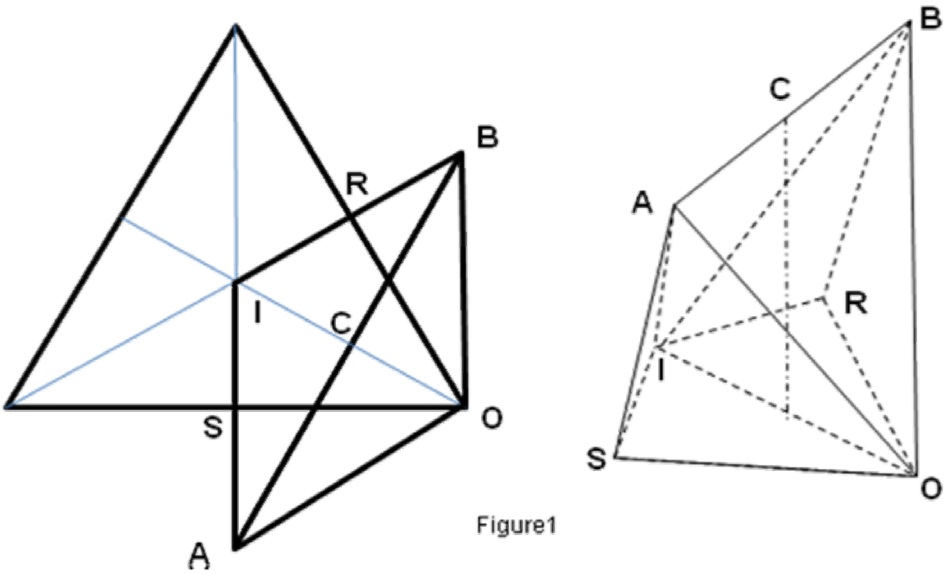

Figure1

The element of Siro is on the basis representing third of the area of a side of an elementary tetrahedron having form of a symmetric quadrangle S I R O with lengths SO = OR = 1/2 K, SI = RI = $\sqrt{3}$ K/6 and right angles at the corner S and R.

The top edge AB we halve and name the central point C.

Also it is possible to allocate the following pyramids SROC SIRC SOAC SIAC, ROCB, IRCB. Pair of pyramids SOAC and ROCB and also pair SIAC and IRCB have identical volume owing to symmetry of the element of Siro. All these pyramids are separately shown on figure 2,3,4,5,6,7

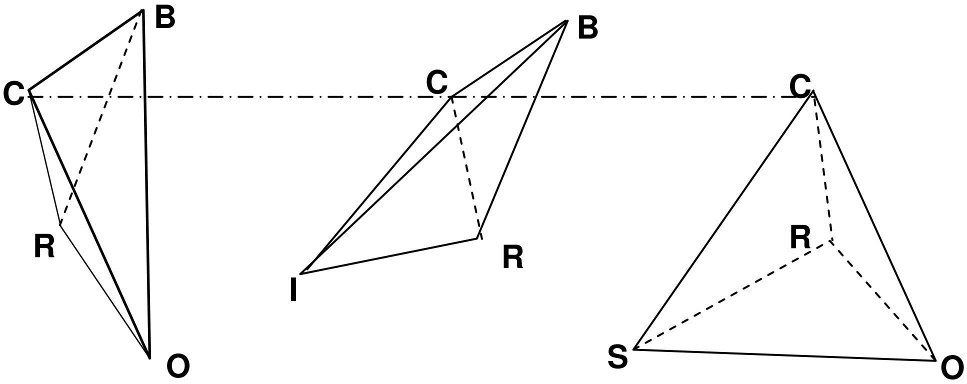

Figure 2 Figure 3 Figure 4

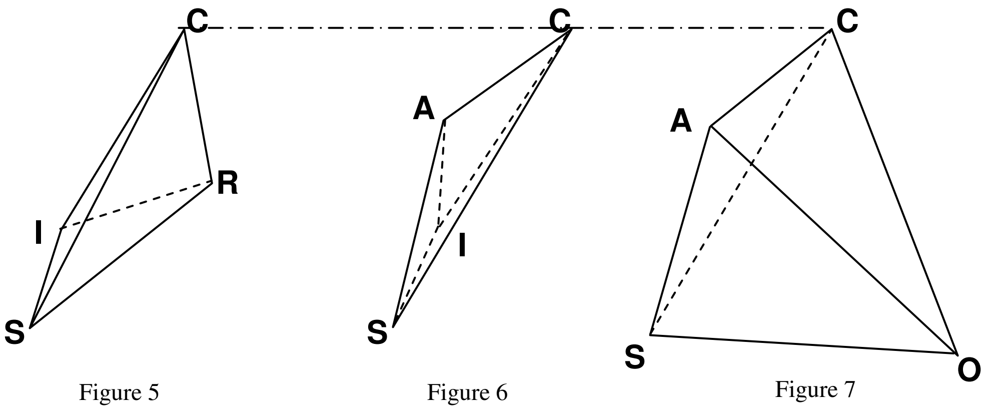

Figure 5 Figure 6 Figure 7

The volume of pyramids SROC and SIRC is calculated in the traditional way under the formula 1

$$\mathbf{V} = \frac{1}{3} \cdot \mathrm{S} \cdot \mathrm{h} \qquad \textbf{Formula 1}$$

where

V – the volume of a pyramid
S – the area of a pyramid base
h – the height of a pyramid

Thus, the joint volume of these two pyramids is equal to third of volume of an element of Siro.
Recognizing that the area of basis SRO is three times more than the area of SRI under formula 1 we get: the volume of pyramid SROC is equal 1/4 volume of an element of Siro and the volume of pyramid SIRC is equal 1/12 volume of an element of Siro.

I apply the formula of volumes deduced by me for this case to calculation of volumes of pyramids SOAC and SIAC
For drawing up this formula we will consider a pyramid as to stem from crossed straight lines which each end is connected to two ends of other straight line. Always it is possible to choose such position that crossed straight lines are in parallel planes
At first we will consider a case 1 of the straight lines crossed at right angle as it is shown in figure 8.

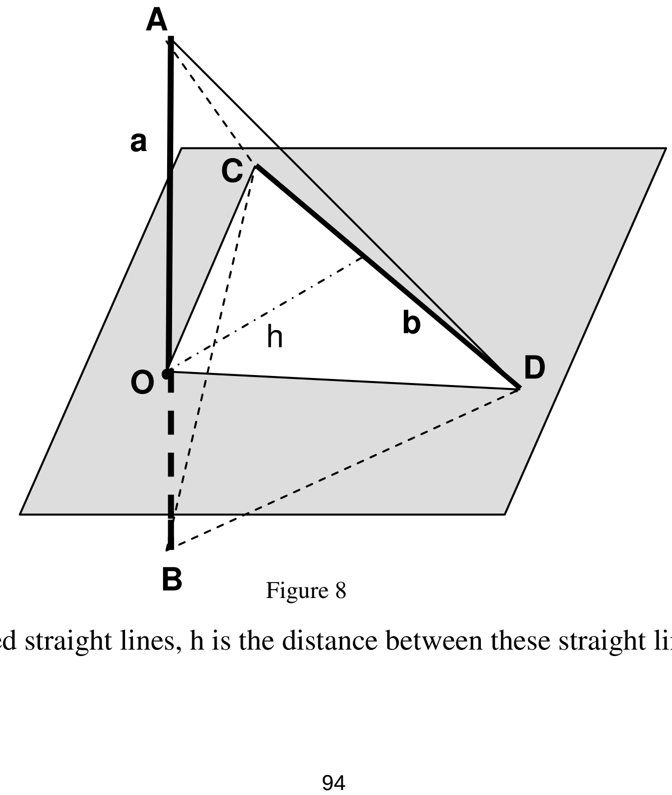

Figure 8

Here a and b are crossed straight lines, h is the distance between these straight lines.

In drawing it is possible to see pyramid AOCD and pyramid BOCD. Their volume is possible to calculate on formula 1.
Pieces AO and OB are in the sum equal to the length of a.
For calculation of volume of two these pyramids I offer the formula 2

$$\mathbf{V} = \frac{a \cdot b \cdot h}{6}$$ **Formula 2**

But generally crossed straight lines a and b have not a right angle like in figure 9

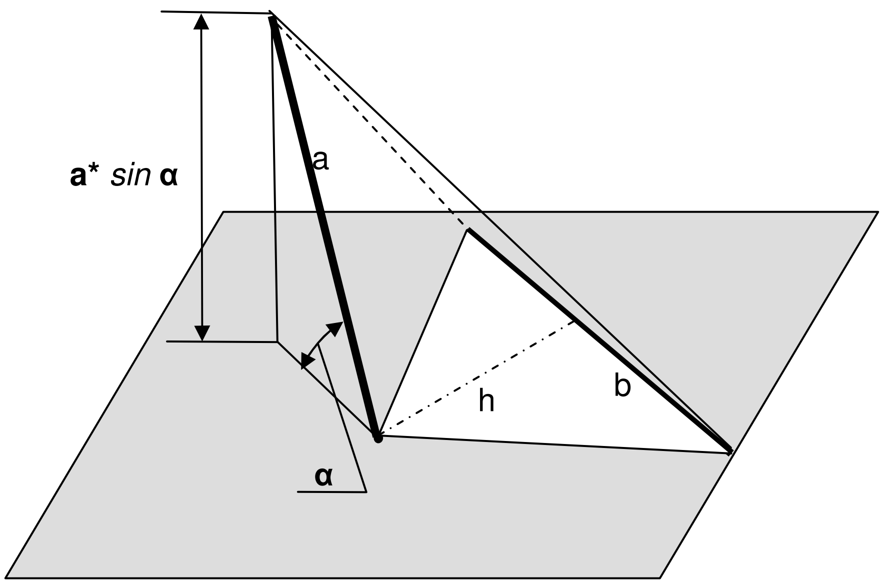

Figure 9

In this case 2 it is necessary to bring value of the angle between the straight lines a and b in the formula. So we will receive the formula 3

$$\mathbf{V} = \frac{a \cdot b \cdot \mathit{sin}\angle ab}{6}$$ **Formula 3**

I named it formula of Siro for calculation of volumes.
In this formula h is the distance between straight lines a and b.

- If the distance is equal 0 than the straight lines are transversal and the volume is equal 0. Pyramids are not present.
- If the angle is equal 0 than the straight lines are parallel and the volume is equal 0 too. Pyramids are not present

We use this formula for calculation of volumes of pyramids ACSO and ACSI
The distances h of the straight line AC to the straight lines SO and SI are equal.
For the pyramid ACSO (figure 10) we will consider the crossed straight lines AC and SO. They are equal among themselves and equal 1/2 K. The angle between them is equal 60º, see figure 10.
The volume of pyramid ACSO under the formula 3 will make

$$\frac{K/2 \cdot K/2 \cdot h \cdot \mathit{sin}60^{o}}{6} = \frac{h \cdot K^2\sqrt{3}}{48}$$ **Formula 4**

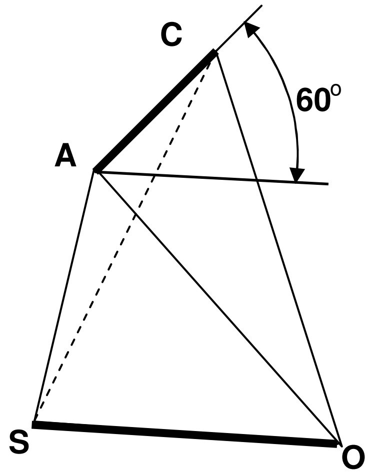

Figure 10

For the pyramid ACSI (figure 11) we will consider the crossed straight lines AC, its length is equal 1/2K, and SI, its length is equal to $\sqrt{3}$ K/6 and the angle between them is equal 30º, see figure 11

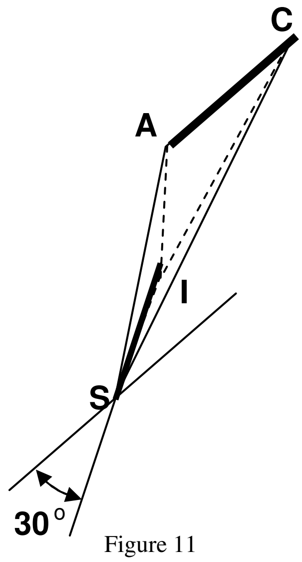

Figure 11

The volume of pyramid ACSI under the formula 3 will make

$$\frac{\mathrm{K}/2\cdot\frac{\mathrm{K}\sqrt{3}}{6}\cdot\mathrm{h}\cdot sin30^{\circ}}{6}=\frac{\mathrm{h}\cdot\mathrm{K}^2\sqrt{3}}{144}$$ **Formula** 5

The sum of volumes of pyramids ACSO and ACSI is equal

$$\left(\frac{\mathrm{K}^2\sqrt{3}}{144}+\frac{\mathrm{K}^2\sqrt{3}}{48}\right)\cdot\mathrm{h}=\frac{\mathrm{K}^2\sqrt{3}}{36}\cdot\mathrm{h}$$ **Formula 6**

The total volume of pyramids ACSO and ACSI should make third of all volume of an element of Siro. Third of volume of an element of Siro according to **[1]** is equal $\frac{\mathrm{K}^3\sqrt{2}}{36}$.

Thus the height of an element of Siro h is equal $\frac{\mathrm{K}\sqrt{2}}{\sqrt{3}}$

I have received the formula to calculate a pyramid volume if it is presented by pair of crossed straight lines and I have shown into what pyramids it is possible to break an element of Siro. Thus I have checked up the formula and broken an element of Siro into six pyramids. Volumes of pyramids CBRO, CSOR, CASO are equal to a quarter of the volume of an element of Siro. Volumes of pyramids CBIR CRIS ACSI are equal 1/12 volume of an element of Siro.
Certainly offered formula of volumes can be used also in other areas of calculations. It is possible to present any difficult form by a file of points, which it is possible to unite in pyramids and thus to calculate body volume.

References

[1] Transition from tetrahedron to octahedron with the help of SIRO element
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