

# Visual representation of the power functions of the second and third degrees

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**Summary.** We may look at our world from different points of view to see things, already known, in an absolutely new way depending on our point of view. In the following article we are going to consider a sequence of numbers raised to a power determined by an integer. In the following article we are going to consider functions of the second and third powers. It is offered to single out elementary figures and bodies, from which, as from children bricks it is possible to build a model of the power function of the second and third degrees. Among bodies, using which it is possible to create model of the power function of the third degree there exists the body named by the author an “**Siro element**”. Considering the law of the changing the bodies quantity from which the model of the power function is constructed, we have got the formula allowing to receive any element of the sequence of numbers, raised to the second or third degrees.

**Designations**  
 $1 * 2 * 3 * 4 * 5 = 120 = 5!$       **!** - Factorial  
 $1 + 2 + 3 + 4 + 5 = 15 = 5 I$       **I** - the sum of a natural numbers row

## 1 Representation

In the given article the sequence of numbers raised to the second and the third degrees is considered. At first for reception of the more complete picture the sequence of numbers raised to the fourth power is also considered. In each of sequences we will take away from each subsequent member the previous, one receiving a new sequence. In each newly received sequence we will perform the above-mentioned operation till getting the subsequence of equal numbers. This process is shown in **Tables 1,2,3**

1, 4, 9, 16, 25, 36, 49
3 5 7 9 11 13
2 2 2 2 2

**Table 1:** A constant difference for the **second** degree

1	8	27	64	125	216	343	512	729
7	19	37	61	91	127	169	217	
	12	18	24	30	36	42	48	
		6	6	6	6	6	6	

**Table 2:** A constant difference for the **third** degree

1	16	81	256	625	1296
	15	65	175	369	671
		50	110	194	302
			60	84	108
			24	24	

**Table 3:** A constant difference for the **fourth** degree

The constant number	for the second power is	<b>2</b>	$1*2=2$
	for the third power -	<b>6</b>	$1*2*3=6$
	for the fourth power -	<b>24</b>	$1*2*3*4=24$ .

This law can be presented by **Formula 1**.

$$CD_n = n!$$

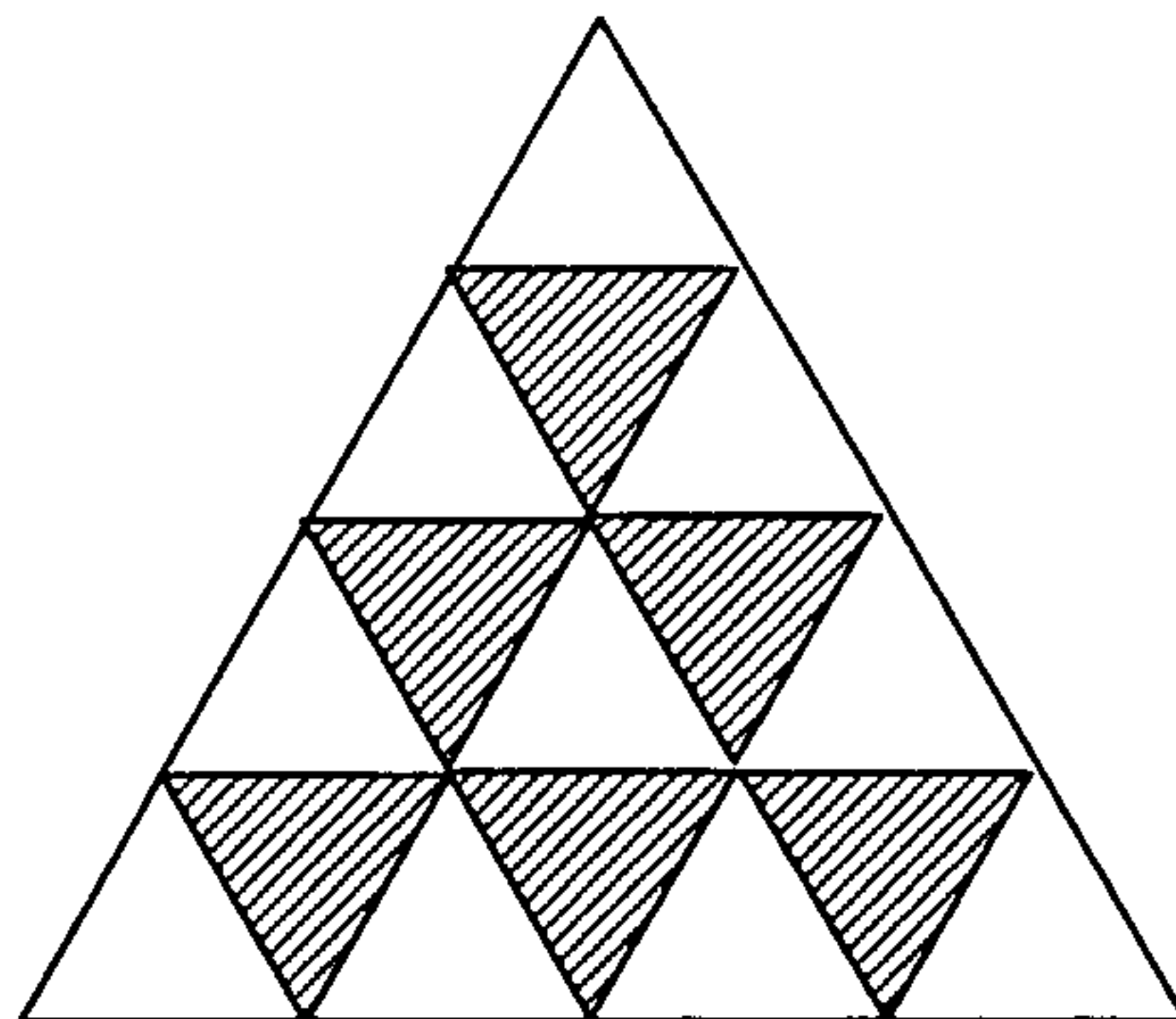
**CD** - Constant **D**ifference

**Formula 1:** Definition of a constant difference for any power

## 2 The graphic model of power function of the second degree

Let's consider a sequence of numbers raised to the second power from the point of view of the graphic representation of this dependence. Among many other possible variants the variant submitted in figure 1 is offered. The triangle, divided in such a way by direct lines, enables to present graphically the sequence shown in table 1.

The elementary triangles, the number of which corresponds to the meaning of number raised to the second power, have two directions of orientation of the tops - upwards tops and downwards tops. The triangles with the downwards top in **Figure 1** are shown shaded.



**Figure 1:** Graphic model of power function of the second degree

In **Table 4** it is shown how the amount of these elementary triangles changes at the transition from one number raised to the second power to another.

<b>N</b>	1	2	3	4	5	6	7	8	9
$n^2$	1	4	9	16	25	36	49	64	81
<b>Up Top</b>	1	3	6	10	15	21	28	36	55
<b>Down Top</b>	0	1	3	6	10	15	21	28	36

**Table 4:** Change of the quantity forming the graphic model of power function of the second degree

This law can be demonstrated by **Formula 2**

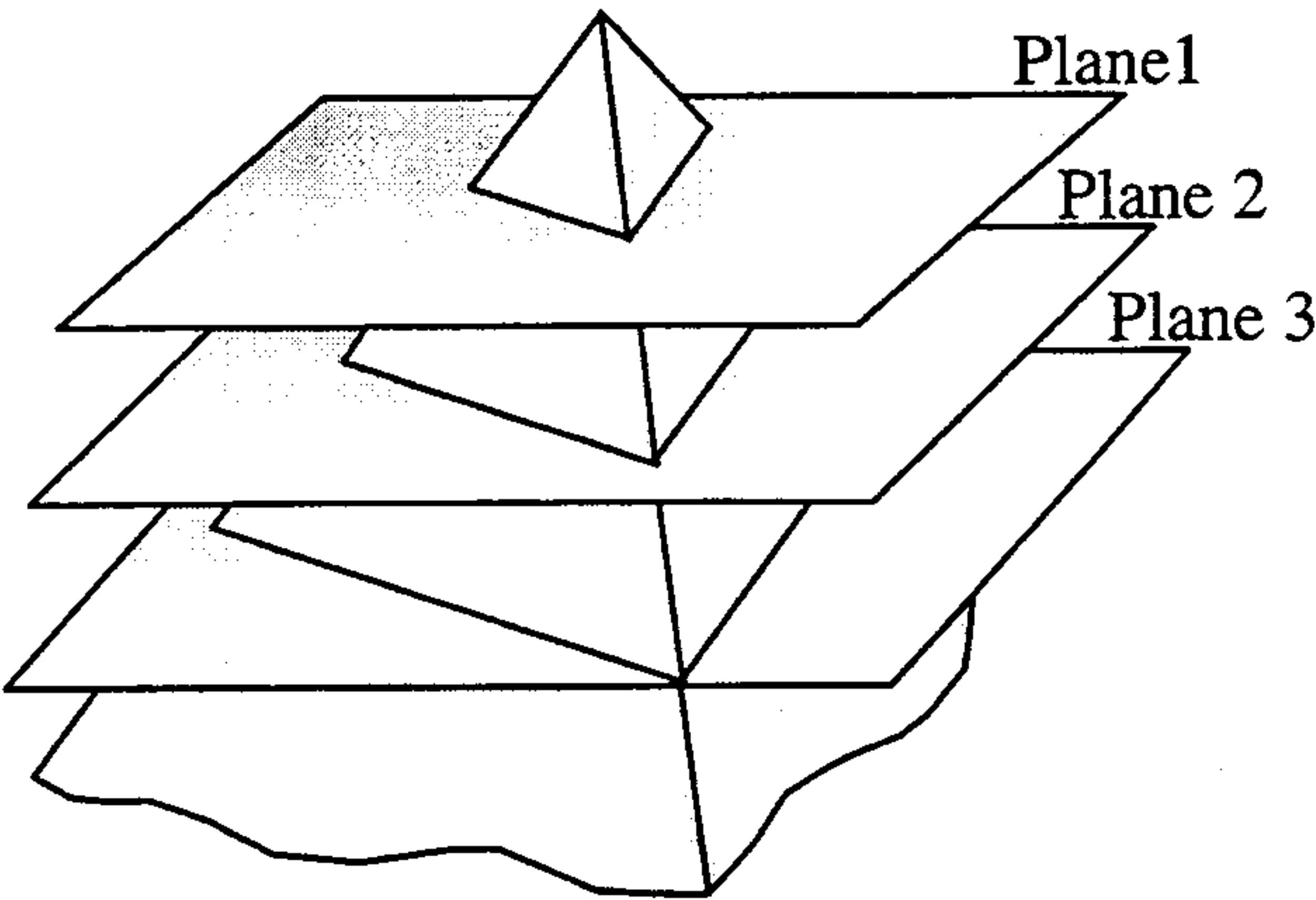
$$n^2 = nI + (n-1)I$$

**Formula 2:** Definitions of any number of an integer's sequence, raised to the second degree



### 3 The graphic model of power function of the third degree

Now we will consider a sequence of numbers raised to the third power and offer the graphic design of this sequence. Such model can be presented as a rectilinear three-edged pyramid. **(Figure 2)** To present elementary volumes, the number of which corresponds to the sequence of numbers raised to the third degree, let's divide this pyramid by the family of the planes, perpendicular to the axis of this pyramid and removed from each other the same distance.



**Figure 2:** The scheme of formation the spatial model of power function of the third degree

First of these planes will separate one elementary pyramid with the top focused upwards.

The second plane will separate the second layer of 7 elementary volumes that together with the first pyramid corresponds to number 2 raised to the third power.

The third plane will separate a layer consisting of 19 elementary volumes, that together with the first pyramid and the second layer corresponds to number 3 raised to the third power, and so on.

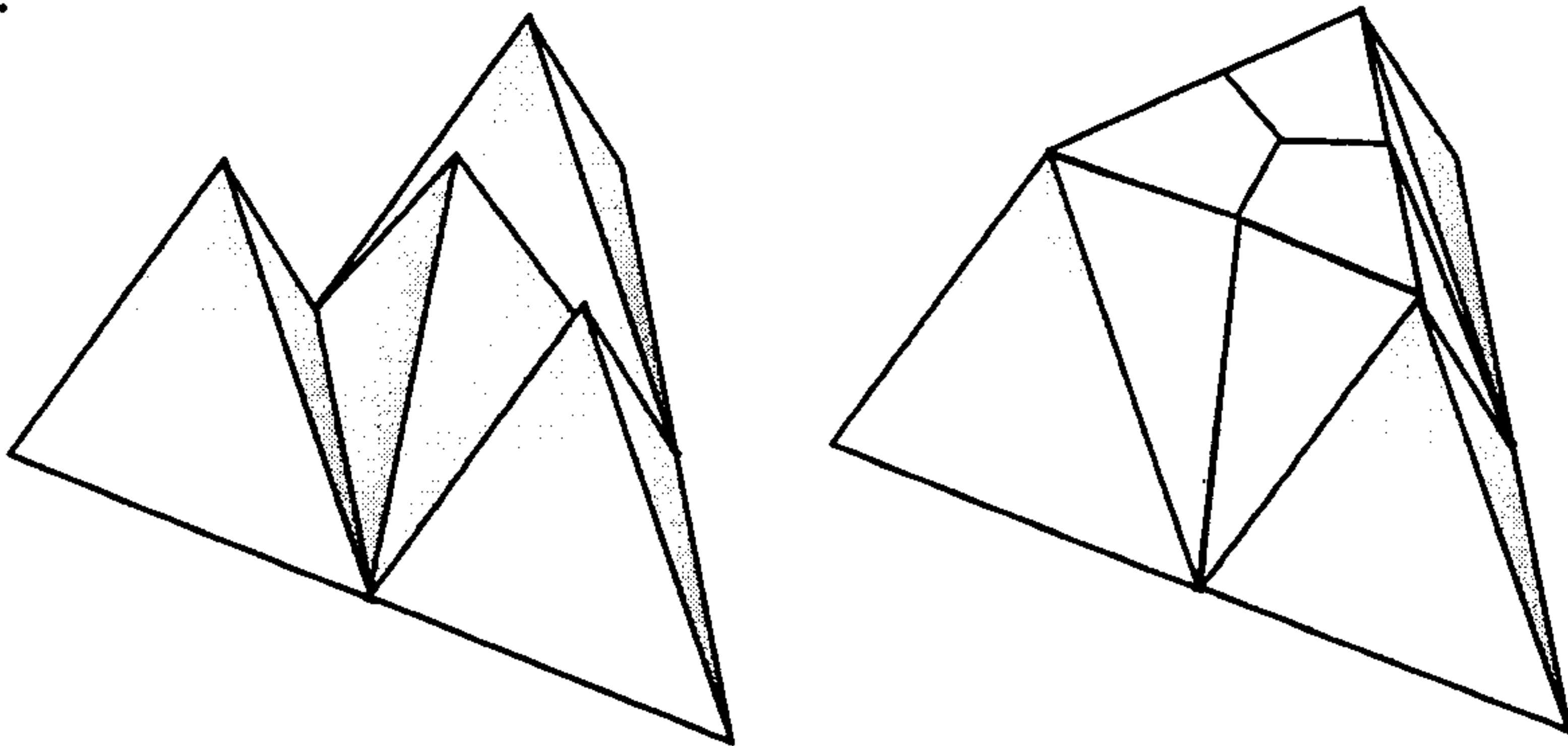
Thus, between the first and second planes there will be 7 elements of the same volume.

Between the second and third planes there will be 19 elements of the same volume.

The quantity of these elements corresponds to a sequence of table 2, submitted in the second line

Now let's consider, what elements each layer consists of.

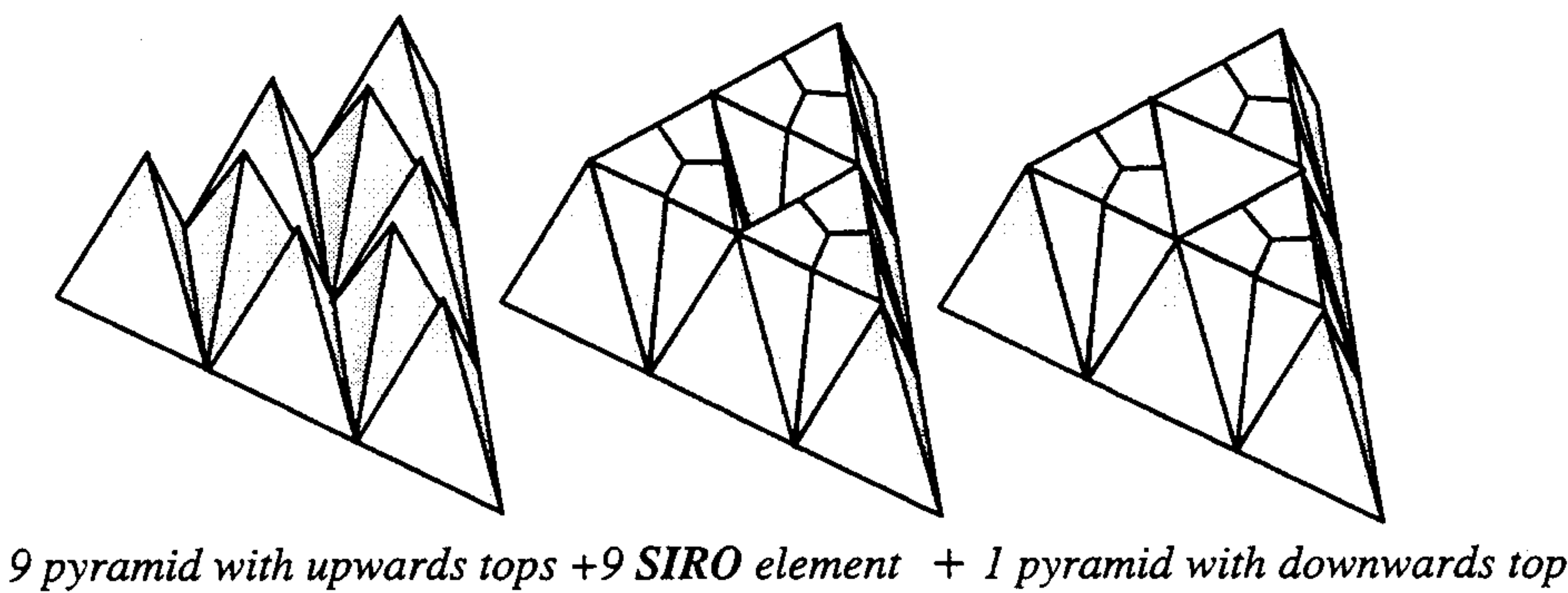
As it has been already mentioned, there are 7 elements between the first and second planes. Among them: four pyramids with upwards focused tops, just the same, as a pyramid separated by the first plane, and three SIRO elements. **(Figure 3)**, the information about them is given below.



4 pyramid with upwards tops + 3 SIRO element

**Figure 3:** Formations of the second layer of the spatial model of the third degree power function

Between the second and third planes there are 19 elements. Among them: 9 pyramids with upwards focused top, one pyramid with a downwards-focused top and 9 SIRO elements (**Figure 4**).



**Figure 4:** Formations of the third layer of the spatial model of the third degree power function

There are 37 elements of the same volume between the third and fourth planes. Among them: 16 pyramids with upwards focused tops, 3 pyramids with downward focused tops and 18 SIRO elements.

Thus, the whole model can be constructed from three kinds of elements of the same volume.

- Pyramid with an upwards focused top
- Pyramid with a downwards focused top
- SIRO element

It should be mentioned that for creation the geometrical model of the second degree power function, we used a flat, two-dimensional model, and for power function of the third degree - a three-dimensional model.

In **Table 5** it is shown how the quantity of three allocated elements changes at transition from a layer to layer, that corresponds to transition from one number raised to the third degree to another one

N	1	2	3	4	5	6	7	8	9	10	11
Up Top	1	4	9	16	25	36	49	64	81	100	121
Down Top			1	3	6	10	15	21	28	36	45
SIRO		3	9	18	30	45	63	84	108	135	165
$\Sigma_1=UT+DT+S$		7	19	37	61	91	127	169	217	271	331
$n^3=(n-1)^3+\Sigma_1$	1	8	27	64	125	216	343	512	729	1000	1331

**Table 5:** Change of the quantity of elements forming the spatial model of the third degree power function

**Formulas 3 and 4** show the results represented in table 5.

$$\delta n^3 = n^2 + (n - 2)I + 3 * (n-1)I$$

**Formula 3:** Definition of quantity of elements in each layer of spatial model of the third degree power function

$$n^3 = (n - 1)^3 + n^2 + (n - 2)I + 3 * (n-1)I$$

**Formula 4:** Definitions of any number of an integers sequence, raised to third degree



4 SIRO element

In **Figures 5a,5b,5c** it is shown how the SIRO Element and the ratio of its geometrical elements are being formed depending on the length of an elementary pyramid

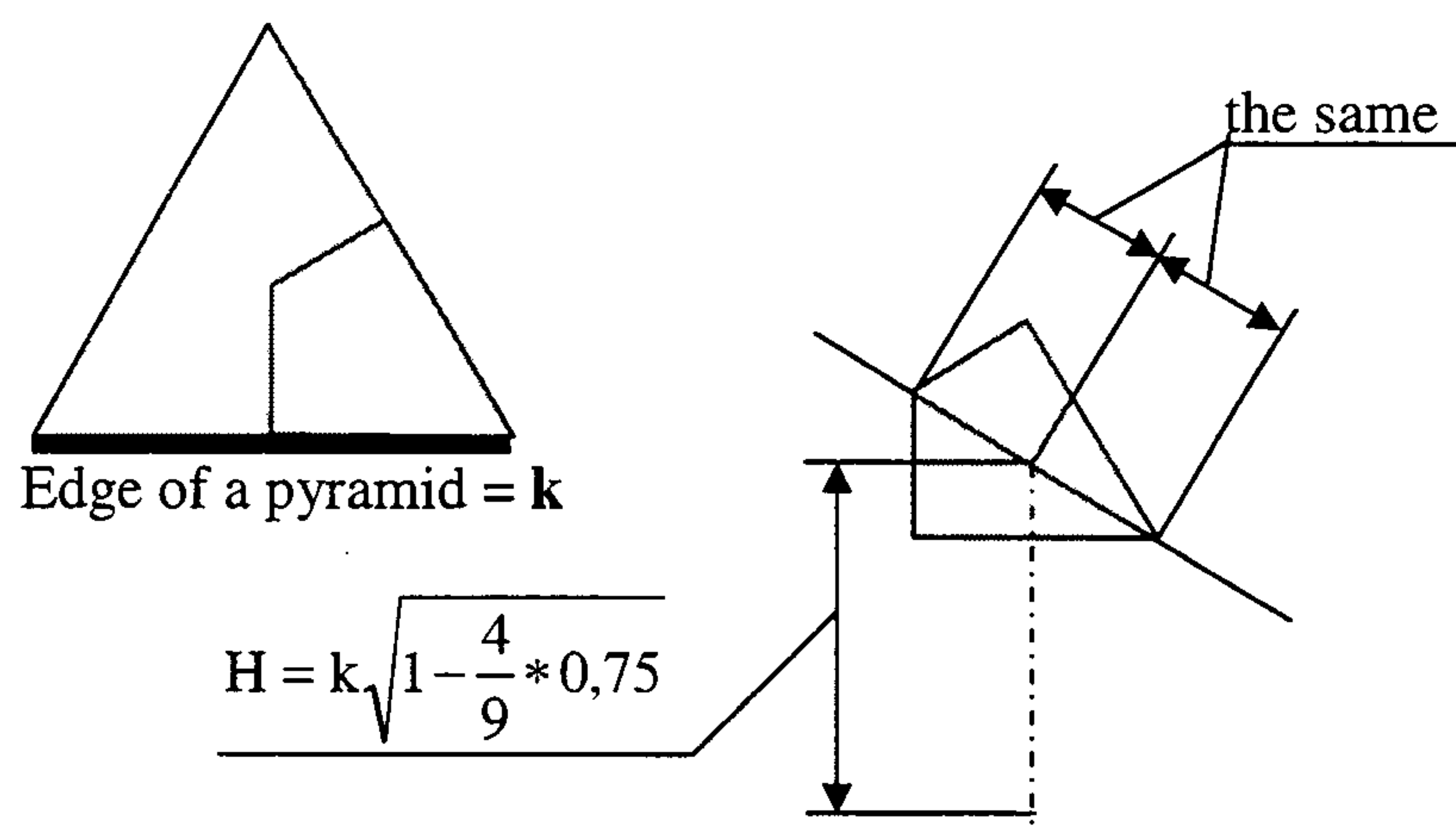


Figure 5a: Geometrical ratio of **SIRO** element

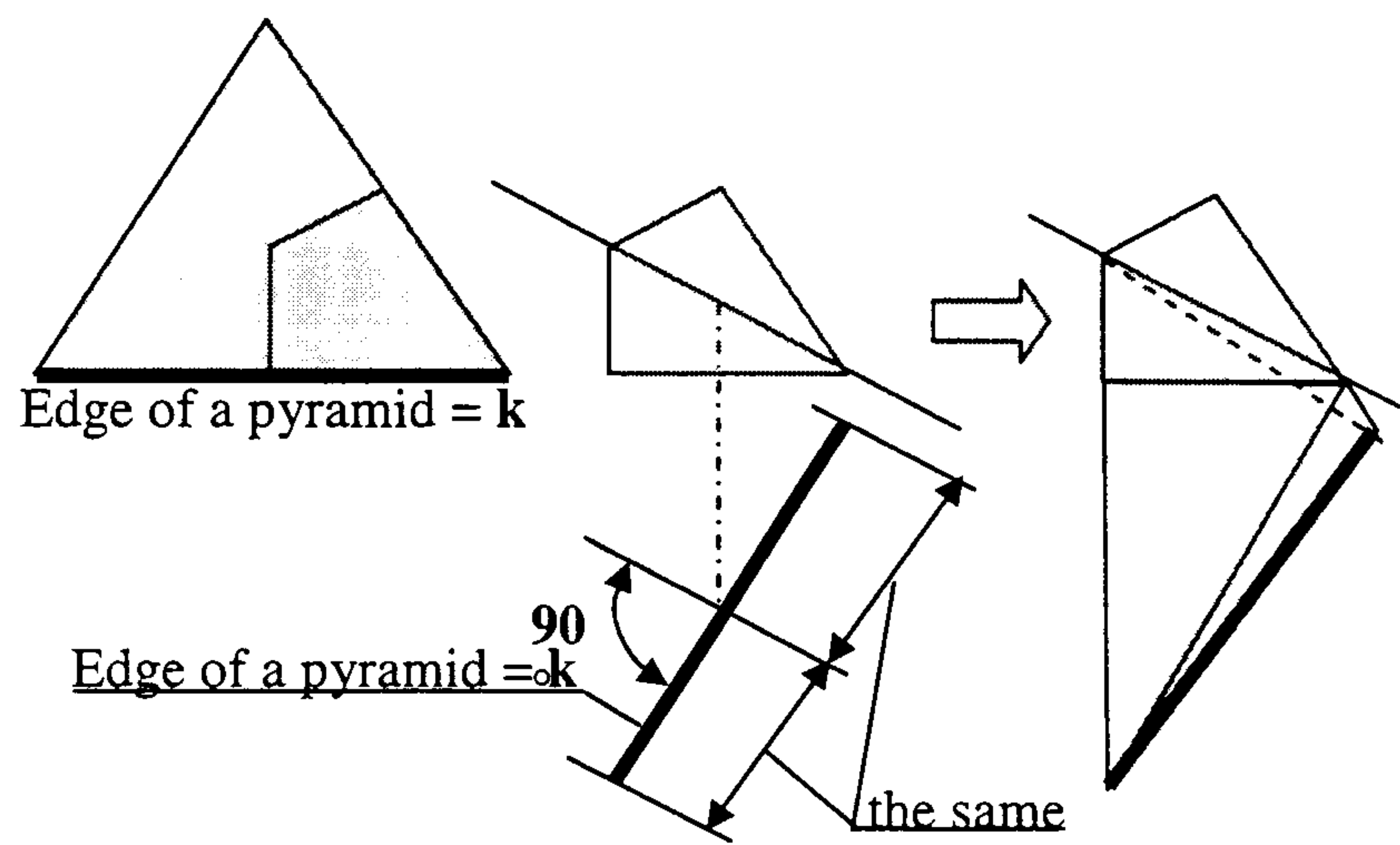


Figure 5b: Geometrical ratio of **SIRO** element

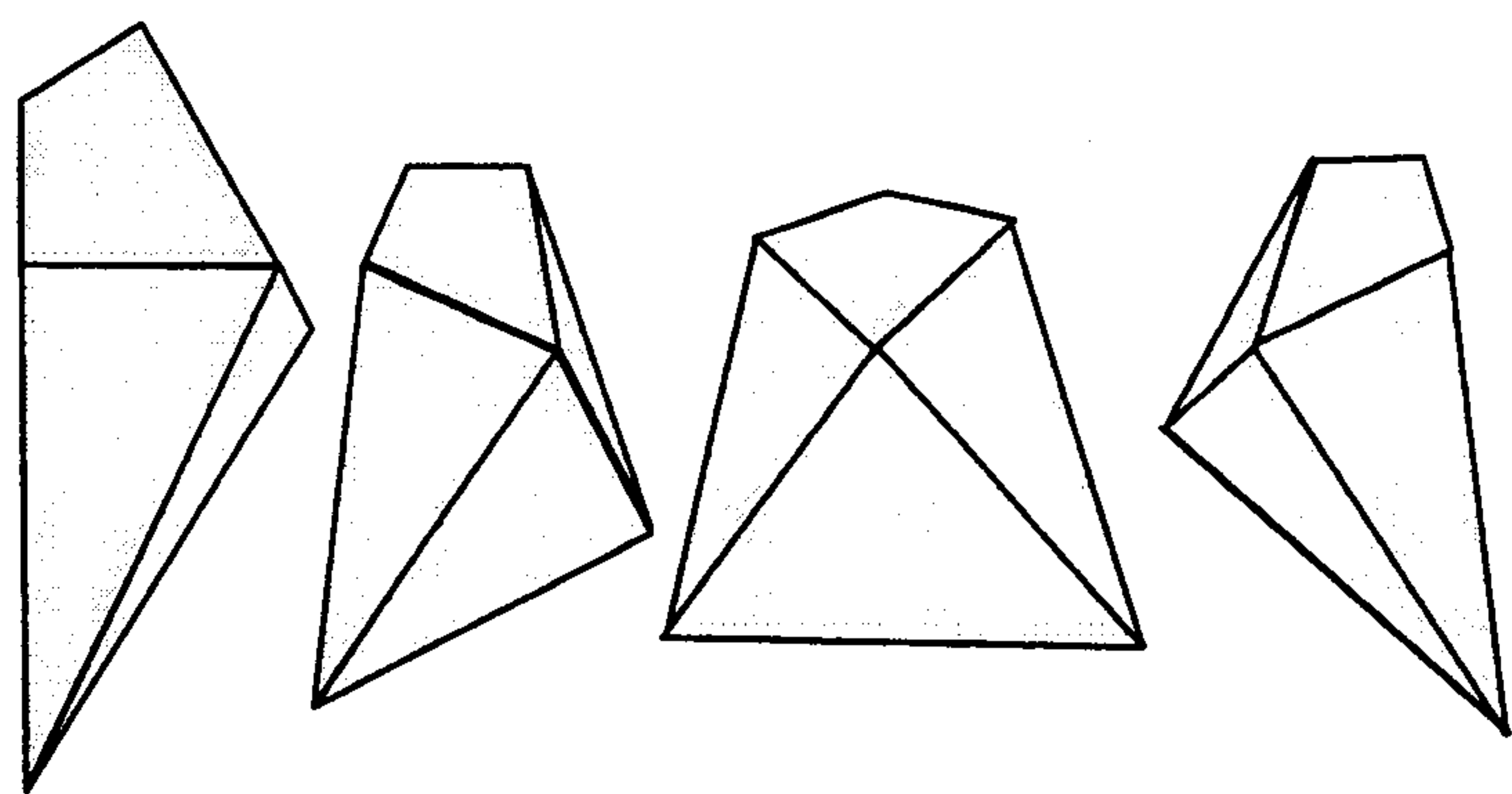
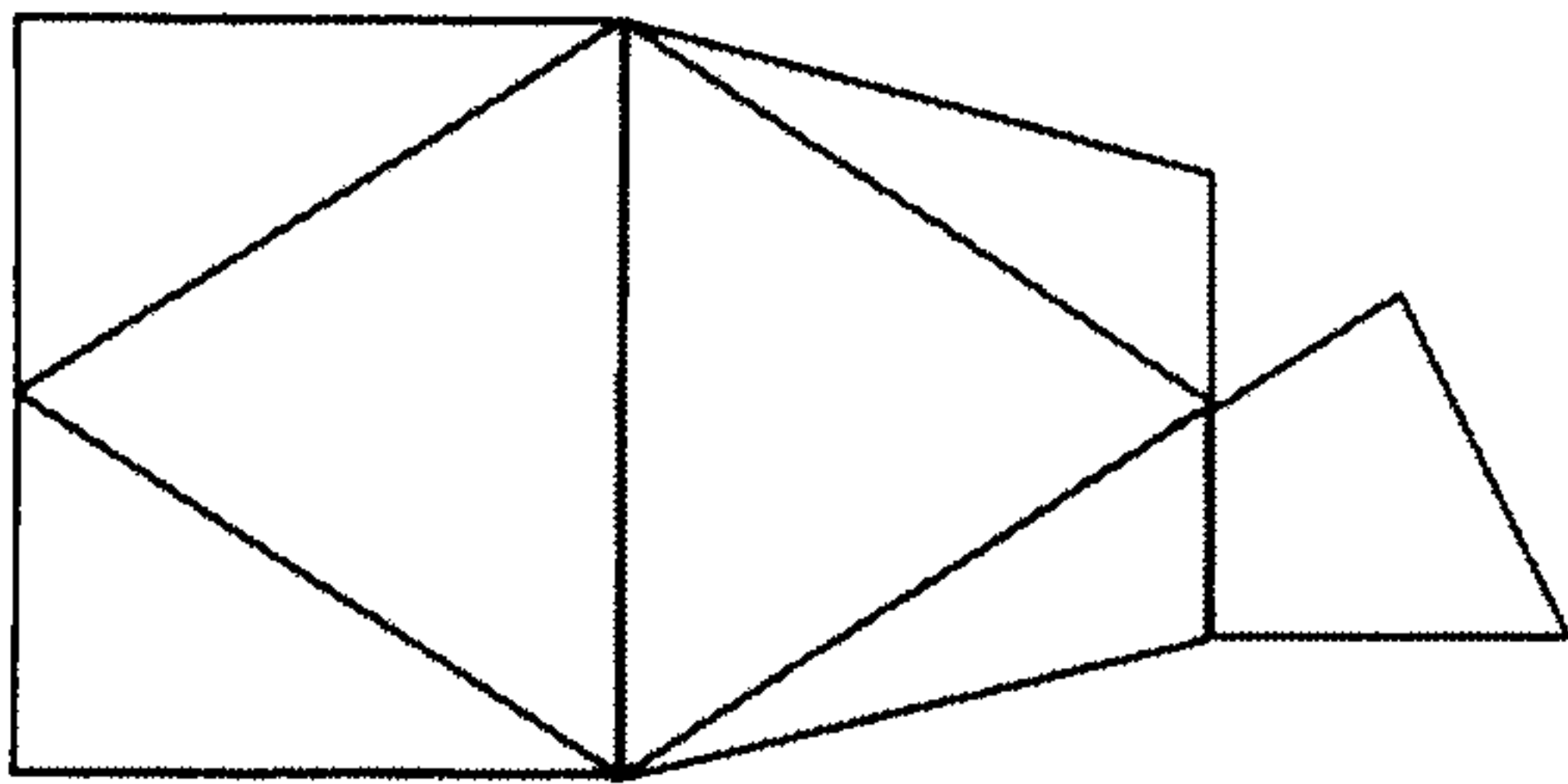


Figure 5c: **SIRO** element itself

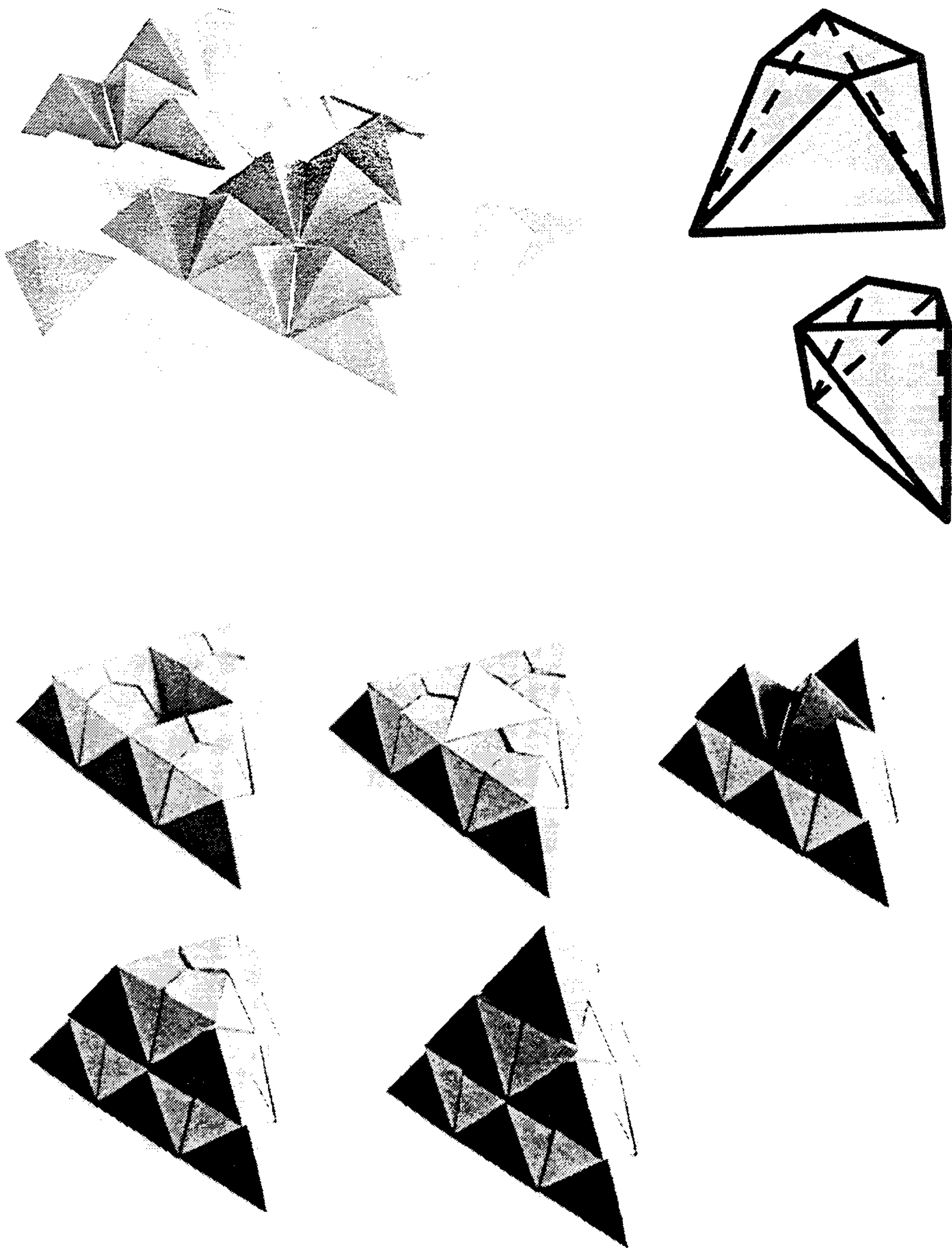
In **Figure 6** the development of this element is submitted.



**Figure 6:** The **SIRO** element development

Other properties of this element will be described in the following works.

**Appendix:** the photo of the model of the third degree power function and SIRO element



## Conclusion

In the given research there having introduced formulas making it possible to get any member of a sequence of natural number raised to the second or third degrees. There was described SIRO element – one of the elements, which may be used creating a spatial model of a sequence of natural numbers raised to the third degree.

The researches performed by the author allow to hope that it is possible to deduce formulas similar to the formulas of the second and thirds degrees for higher powers too.

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