Geometrical characteristics of SIRO element. Transition from tetrahedrons to octahedron with the help of SIRO elements

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ABSTRACT We may look at our world from different points of view To see things, already known, in an absolutely new way depending on our point of view. In the following article we are going to consider a sequence of numbers raised to a power determined by an integer. In the following article we are going to consider functions of the second and third powers. It is offered to single out elementary figures and bodies, from which, as from children bricks it is possible to build a model of the power function of the second and third degrees. Among bodies, using which it is possible to create model of the power function of the third degree there exists the body named by the author an «SIRO element ".Considering the law of the changing the bodies quantity from which the model of the power function is constructed, we nave got the formula allowing to receive any element of the sequence of numbers, raised to the second or third degrees.

Keywords: Element SIRO as a part of model the power function of the third degree. Transition from tetrahedrons to octahedron with the help of SIRO elements

DESIGNATIONS

$1*2*3*4*5 = 120 = 5!$! - Factorial

$1+2+3+4+5 = 15 = 5\mathbf{I}$ **I** - the sum of a natural numbers row

1. REPRESENTATION

In the given article the sequence of numbers raised to the second and the third degrees is considered. At first for reception of the more complete picture the sequence of numbers raised to the fourth power is also considered.

In each of sequences we will take away from each subsequent member the previous, one receiving a new sequence. In each newly received sequence we will perform the above-mentioned operation till getting the subsequence of equal numbers.

This process is shown in **tables 1,2,3**

1,	4,	9,	16,	25,	36,	49
3	5	7	9	11	13	
2	2	2	2	2		

Table 1: A constant difference for the **second** degree

1	8	27	64	125	216	343
7	19	37	61	91	127	
12	18	24	30	36		
6	6	6	6			

Table 2: A constant difference for the **third** degree

1	16	81	256	625
15	65	175	369	
50	110	194		
60	84			
24				

Table 3: A constant difference for the **fourth** degree
The constant number

for the second power is **2** 1*2=2
for the third power - **6** 1*2*3=6
for the fourth power - **24** 1*2*3*4=24
This law can be presented by **formula 1**

$$CD_n = n!$$

CD - Constant Difference

Formula 1: Definition of a constant difference for any power

2. THE GRAPHIC MODEL OF POWER FUNKTION OF THE SECOND DEGREE

Let's consider a sequence of numbers raised to the second power from the point of view of the graphic representation of this dependence. Among many other possible variants the variant submitted in figure 1 is offered. The triangle, divided in such a way by direct lines, enables to present graphically the sequence shown in table 1.

The elementary triangles, the number of which corresponds to the meaning of number raised to the second power, have two directions of orientation of the tops - upwards tops and downwards tops.

The triangles with the downwards top in **Figure 1** are shown shaded.

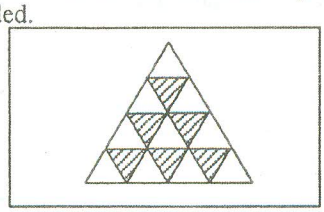

Figure 1: Graphic model of power function of the second degree

In **table 4** it is shown how the amount of these elementary triangles changes at the transition from one number raised to the second power to another.

n	1	2	3	4	5	6
n^2	1	4	9	16	25	36
Up Top	1	3	6	10	15	21
Down Top	0	1	3	6	10	15

Table 4: Change of the quantity forming the graphic model of power function of the second degree

This law can be demonstrated by **formula 2**

$$n^2 = nI+(n-1)I$$

Formula 2: Definitions of any number of an integers sequence, raised to the second degree

3. THE GRAPHIC MODEL OF POWER FUNCTION OF THE THIRD DEGREE

Now we will consider a sequence of numbers raised to the third power and offer the graphic design of this sequence. Such model can be presented as a rectilinear three-edged pyramid. **(Figure 2)** To present elementary volumes, the number of which corresponds to the sequence of numbers raised to the third degree, let's divide this pyramid by the family of the planes, perpendicular to the axis of this pyramid and removed from each other the same distance.

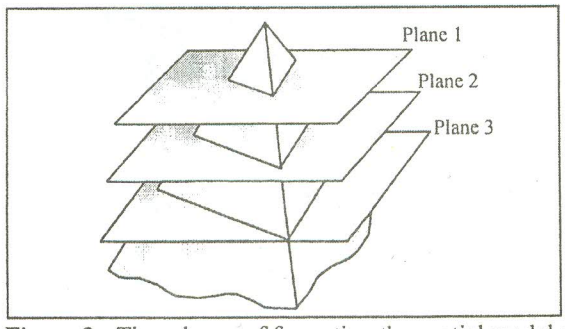

Figure 2: The scheme of formation the spatial model of power function of the third degree

First of these planes will separate one elementary pyramid with the top focused upwards.

The second plane will separate the second layer of 7 elementary volumes that together with the first pyramid corresponds to number 2 raised to the third power.

The third plane will separate a layer consisting of 19 elementary volumes, that together with the first pyramid and the second layer corresponds to number 3 raised to the third power, and so on.

Thus, between the first and second planes there will be 7 elements of the same volume.

Between the second and third planes there will be 19 elements of the same volume

The quantity of these elements corresponds to a sequence of table 2, submitted in the second line

Now let's consider, what elements each layer consists of.

As it has been already mentioned, there are 7 elements between the first and second planes. Among them: four pyramids with upwards focused tops, just the same, as a pyramid separated by the first plane, and three SIRO elements. **(Figure 3)**, the information about them is given below.

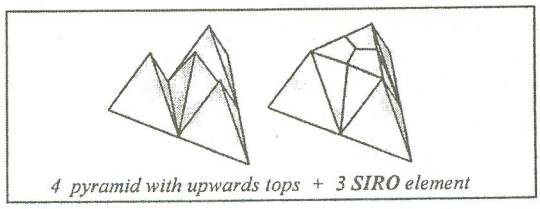

Figure 3: Formations of the second layer of the spatial model of the third degree power function

Between the second and third planes there are 19 elements. Among them: 9 pyramids with upwards focused top, one pyramid with a downwards-focused top and 9 SIRO elements **(Figure 4)**.

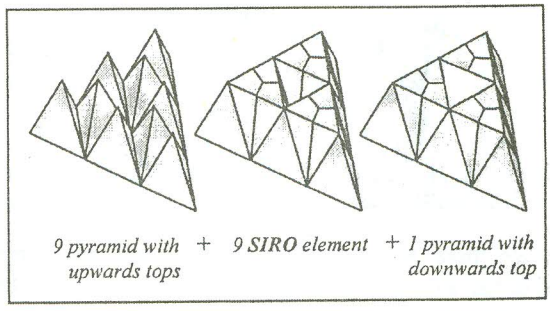

Figure 4: Formations of the third layer of the spatial model of the third degree power function

	1	2	3	4	5	6
Up Top	1	4	9	16	25	36
Down Top			1	3	6	10
SIRO		3	9	18	30	45
Σ_1=UT+DT+S		7	19	37	61	91
$n^3=(n-1)^3+\Sigma_1$	1	8	27	64	125	216

Table 5: Change of the quantity of elements forming the spatial model of the third degree power function

Formulas 3 and **4** show the results represented in table **5**.

$$\Sigma_1 = \delta n^3 = n^2 + (n-2)I + 3(n-1)I$$

Formula 3: Definition of quantity of elements in each layer of spatial model of the third degree power function

$$n^3 = (n-1)^3 + n2 + (n-2)I + 3(n-1)I$$

Formula 4: Definitions of any number of an integers sequence, raised to third degree

4. FORMING OF SIRO ELEMENT

Generally sides of **SIRO** element can have various forms. At first we will consider **SIRO** element, which faces the elementary tetrahedron. As a result we receive «a Basic **SIRO** element», which will be described in the following article. All sizes will be given in ratio to the length of the elementary tetrahedron edge-"K"

The formation of **SIRO** element is reasonable to explain, taking into account its arrangement between two elementary tetrahedrons, as shown in **Figure 5a, 5b, 5c, 5d.**

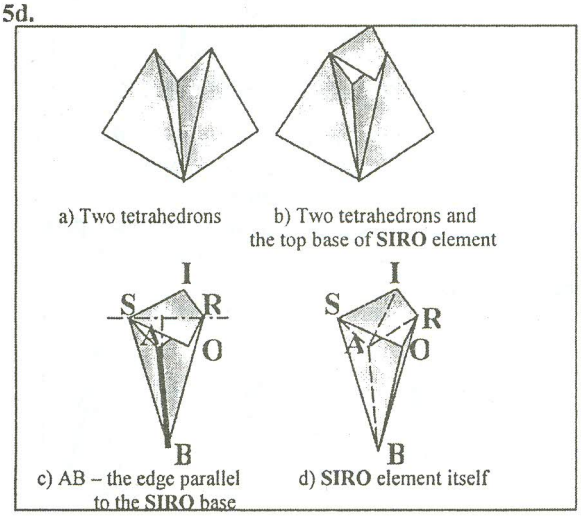

Figure 5:

The top base SIRO is one third of the elementary tetrahedron side and goes parallel to the base of the *elementary tetrahedron. Edge AB is parallel to the top* base SIRO, its length is equal to the length of the elementary tetrahedron edge-"K"

In **Figure 6**. AM=MB; SN=NR; SR ⊥ AB; MN ⊥AB; MN ⊥ SIRO. MN is equal to the height of the elementary tetrahedron. MN may be named as an axis of SIRO element. SIRO element has one plane of symmetry determined by direct lines: SR and MN.

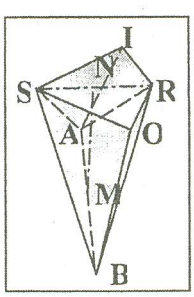

Figure 6: SIRO element

5. GEOMETRICAL CHARACTERISTICS OF SIRO ELEMENT

Certainly, in one small article it's impossible to investigate and to list all properties of **SIRO** element. We will focus only on those ones, which in the ranks of the given article demonstrate the transition from a tetrahedron to an octahedron.

SIRO *element has 7 sides, 11 edges, 6 tops.*

Sides ABS and ABR are absolutely the same as the elementary tetrahedron sides. With these sides the SIRO element faces the appropriate lateral surfaces of the elementary tetrahedrons with the tops focused upwards at creating the volumetric model of the third degree power function.

Sides SOB and SIA (**Figure 7.**) are equal to the half of the elementary tetrahedron side. These sides of SIRO elements face the corresponding surfaces of the elementary tetrahedrons with the tops focused downwards at creating the volumetric model of the third degree power function or are a part of an external surface of the model of the third degree power function

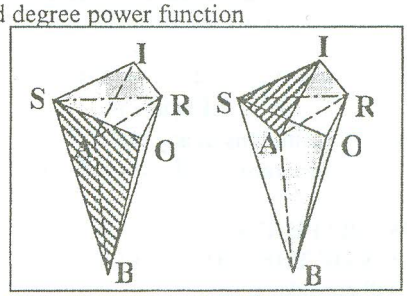

Figure 7: Sides SOB and SIA

Sides AIR and BOR of SIRO element (**Figure 8**) face the other SIRO elements. These sides are perpendicular to the plane of the base of the pyramid of the third degree power function.

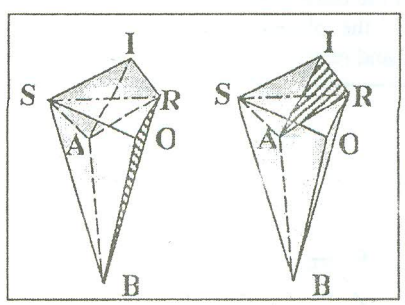

Figure 8: Sides AIR and BOR

The formation of the top side or the top base is shown in **Figure 9**. With these sides SIRO element faces the bottom sides of the elementary tetrahedrons with the tops focused upwards at creating the volumetric model of the third degree power function.

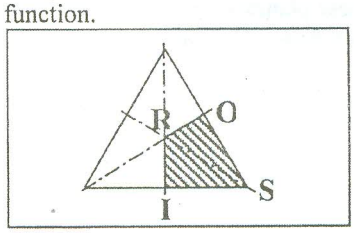

Figure 9: The top side of SIRO element

The edges IS and SB form a plane and are crossed in this plane at right angle, as it is shown in **Figure 10**. It should be mentioned, that angles OSB and OSI are equal 60°

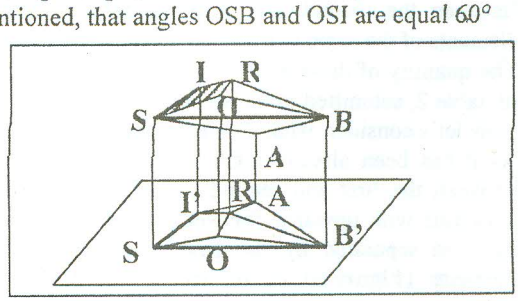

Figure 10: The angle between edges IS and SB

The top S corresponds to the octahedron top. Angle OSB is equal 60°, angle BSA is equal 60°, angle ASI is equal 60°, and angle ISO is equal 60°. The octahedron has just the same angles of the sides at each its top.

6. TRANSITION FROM TETRAHEDRONS TO OCTAHEDRON WITH HELP OF SIRO ELEMENTS

In **Figures 11a, 11в, 11c**, it is consistently shown as adding of three SIRO elements to the elementary tetrahedrons gives the elementary octahedron. The calculation of volumes of these figures (tetrahedrons and octahedron) proves this statement.

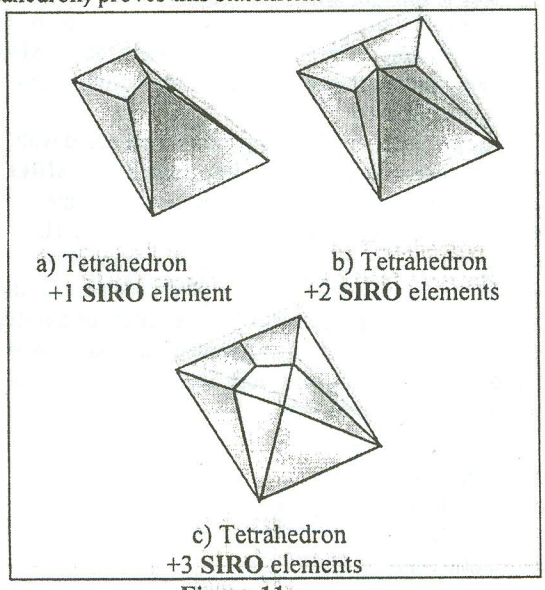

Figure 11c

The sequence of creating the octahedron of three SIRO elements and the elementary tetrahedron is shown **in figures 12a, 12b, 12c, 12d.**

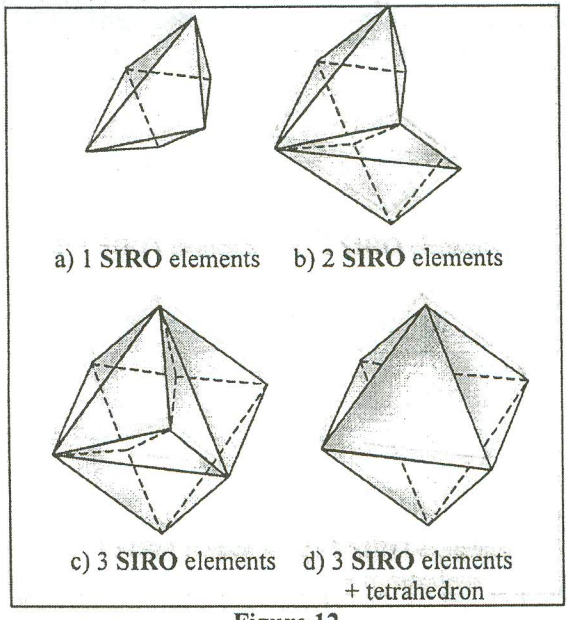

Figure 12

7. VOLUME OF SIRO ELEMENT

The volume of SIRO element and the elementary tetrahedron used for creating the model of the third degree power function are equal. It is obvious, but may be proved in two ways.

For the first way the author uses the formula for calculation a pyramid volume **(Formula 5)**

$$V = \frac{1}{3} \cdot S \cdot h$$

Where

V – the volume of a pyramid

S – the area of a pyramid base

h – the height of a pyramid

Formula 5: Calculation of the pyramid volume

If we take the elementary tetrahedron volume as a unit, the volume of the geometrical model of the first and the second layers together will in 8 times more than the elementary tetrahedron volume **(Formula 6, Figure 13)**

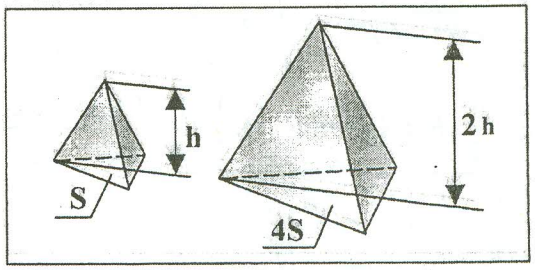

Figure 13: Comparison of the elementary tetrahedron volumes and two layers of the geometrical model of the third degree power function.

$$8\frac{1}{3} \cdot Sosn \cdot h = \frac{1}{3} \cdot 4Sson \cdot 2h$$

Formula 6: Comparisons of sizes of the elementary tetrahedron volumes and two layers of the geometrical model of the third degree power function.

The considering geometrical model has 8 elementary volumes. Among them there are 5 elementary tetrahedrons and 3 **SIRO** elements. It is absolutely clear, that the elementary tetrahedrons volumes, having equal edges - are equal

Three SIRO elements are also equal to each other. That is, volume of SIRO element and the elementary tetrahedron volume, used for creating the geometrical model of the third degree power function, are equal.

For the second method we will also take **Formula 5** and calculate the elementary tetrahedron volume and the elementary octahedron edge, which have the same length, "K"

For the tetrahedron these calculations are submitted by **Formula 7** and **Figure 14.**

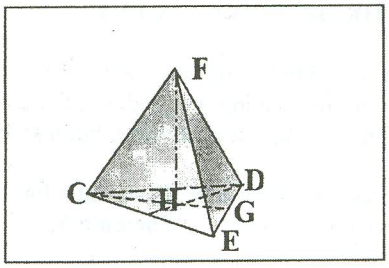

Figure 14: Geometrical characteristics of the elementary tetrahedron, necessary for calculation of its volume

CE = ED = DC = CF = EF = FD = *K*- the length of the elementary tetrahedron edge

$$CG = \sqrt{K^2 - \frac{K^2}{4}} = \frac{K\sqrt{3}}{2}$$

$$CH = \frac{2}{3} \cdot CG = \frac{2}{3} \cdot \frac{K\sqrt{3}}{2} = \frac{K\sqrt{3}}{3}$$

$$FH = \sqrt{CF^2 - CH^2} = \sqrt{K^2 - \frac{3}{9}K^2} = \frac{K\sqrt{2}}{\sqrt{3}}$$

$$V = \frac{1}{3} \cdot \frac{1}{2} \cdot ED \cdot CF \cdot HF = K\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{K\sqrt{3}}{2} \frac{K\sqrt{2}}{\sqrt{3}};$$

$$\boxed{Vtetr = \frac{K^3\sqrt{2}}{12}}$$

Formula 7: Calculations of volume elementary tetrahedron

For the elementary **octahedron** the calculations are represented by **Formula 8**and **Figure 15**

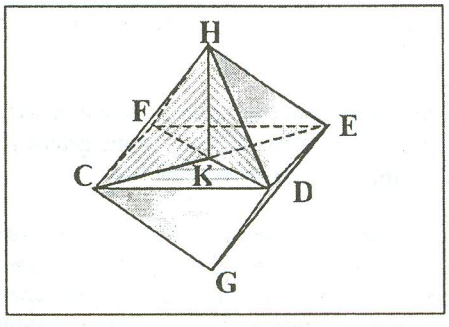

Figure 15: Geometrical characteristics of the elementary octahedron, necessary for calculation of its volume

CD = DE = EF = FC = CH = FH = EH = DH = CG = DG = EG = FG =K- the length of the elementary octahedron edge.

$$CE = \sqrt{K^2 + K^2} = K \cdot \sqrt{2}$$

$$CK = \frac{1}{2} \cdot CE = K \cdot \frac{\sqrt{2}}{2}$$

$$CK = KH = K \cdot \frac{\sqrt{2}}{2}$$

$$Voktaed = 2 \cdot \frac{1}{3} \cdot K^2 \cdot K \cdot \frac{\sqrt{2}}{2}$$

$$\boxed{Voktahed = \frac{K^3\sqrt{2}}{3}}$$

Formula 8: Calculations of the elementary octahedron volume

As it follows from the represented calculations, the octahedron volume is 4 times greater than the tetrahedron volume. That is, equal to each other **SIRO** elements are equal to the elementary tetrahedron volume.

Conclusions

In the process of solving a task connected with the power function visualization, there was found **SIRO** element. In this article attempt of investigating some properties of this geometrical body is undertaken. The interesting enough possibility of transition from one known geometrical body to another gives a hope, that the further researches of properties of this element or abilities of its combination with another geometrical bodies, will permit to elucidate many questions in another spheres of natural sciences in a new way.

8 REFERENCES

Sergei Sirotkin Visual representation of the power functions of the second and third degrees DSG/CK 2003 ISBN:3-86005-394-9 p.317-323

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